Research Article

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Reliability of (1+1) Cascade Model for Weibull Distribution

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Abstract

This study present derives the formula mathematical for reliability cascade model (1+1) for Weibull distribution. Model reliability expressions obtained when Weibull random variables are stress and strength distributions. The ML, Pr, and LS methods estimated the model reliability and used for the comparison between them in simulation with MATLAB using criterion MSE. The comparison indicated that the best estimator was the ML method.

Keywords: Cascade, Parameter, Strength-Stress, Unit, Identically distributed.

Introduction

In the stress-strength model studies, many researchers studied the estimation of the reliability $\mathcal{R} = p(X < Y)$. Redundancy cascade is a hierarchical redundancy system. Consider a special (1+1) Cascade model with units A and B, where one unit A is acting and one unit B is a standby unit. Let X_1 and X_2 refer to strengths of unit (A and B) respectively, suppose that Y_1 and Y_2 refer to conforming stress behaves on them. Now, if the active unit A is failure then the standby unit B is activated, where $X_2 = mX_1$ and $Y_2 = kY_1$, where " k " and " m" denote the stress-strength attenuation factors respectively, such that $0 < m < 1$ and $k > 1$.

Sundar [4] discussed N-cascade system reliability with Weibull Stress-Force distributions, and Weibull cascade system reliability values are measured and graphically displayed. Gogoi and Borah [1] studied two states of reliability, first state supposes that one parameter strength from exponential and two-parameter strength gamma distributions, and the second state assumes that two-parameter are strength exponential and one parameter stress gamma distributions to get the form of reliability 3-cascade system. Umamaheswari and Swathi [5] discussed n-cascade system reliability when stress follows the exponential and strength follows mixed the exponential distributions. Karam and Khaleel [2] studied the formula mathematical of (2+1) cascade Reliability model for generalized inverse Rayleigh random variables. Khaleel and karam [3] studied the formula mathematical of (2+1) cascade Reliability model for inverse Weibull distribution random variables. The main objectives of the present study 1) derive the mathematical formula for the reliability of the (1+1) cascade model with a strength-stress of the Weibull distribution; 2) make a numerical study for the reliability model.

The mathematical formula

Suppose that the strength-stress random variables of two units (one basic and one redundancy standby) to be $X_i \sim W(\sigma, \theta_i)\ ; i = 1,2$ and $Y_j \sim W(\sigma, \rho_j)\ ; j = 1,2$ respectively are independently and identically distributed Weibull with common shape parameter σ and scale parameter $\theta_i ; i = 1,2$ and scale parameter $\rho_j ; j = 1,2$.

The CDF of $W(\sigma,\theta_i)$ is: $F(x) = 1 - e^{-\theta_i x^{\sigma}} \quad x > 0; \sigma, \theta_i > 0 \;; i = 1,2$ **...(1)**

and The CDF of $W(\sigma,\rho_j)$ is: $G(y) = 1 - e^{-\rho y^{\sigma}} \quad y > 0; \sigma, \rho > 0 \;; j = 1,2$ **...(2)**

The PDF of $W(\sigma,\theta_i)$ is: $f(x) = \sigma\theta_i x^{\sigma-1}\, e^{-\theta x^{\sigma}} \quad x > 0; \sigma, \theta_i > 0 \;; i = 1,2$ **...(3)**

and The PDF of $W(\sigma,\rho_j)$ is: $g(y) = \sigma\rho y^{\sigma-1}\, e^{-\rho y^{\sigma}} \quad x > 0; \sigma, \rho > 0 \;; j = 1,2$ **...(4)**

The reliability function for (1+1) cascade model can be formulated as:

$R = R_1 + R_2$ **... (5)**

Case one: when unit A are activated and the unit B acting as the standby unit then :

$R_1 = p[X_1 \geq Y_1]$

$= \int_0^{\infty} p[X_1 \geq Y_1] g(y_1) dy_1$

$= \int_0^{\infty} p[X_1 \geq Y_1] g(y_1) dy_1$

$= \int_0^{\infty} \left[\int_{y_1}^{\infty} f(x_1)\, dx_1\right] g(y_1) dy_1$

$R_1 = \int_0^{\infty} [\bar{F}_{x_1}(y_1)] g(y_1) dy_1$ **... (6)**

substituting equations (1),(2),(3) and (4) in (6), will get as

$$R_1 = \left[\int_0^{\infty} \left(e^{-\theta_1 y_1{}^{\sigma}}\right) \sigma\rho_1 y_1{}^{\sigma-1} e^{-\rho_1 y_1{}^{\sigma}} dy_1\right]$$

$$= \left[\int_0^{\infty} \sigma\rho_1 y_1{}^{\sigma-1} e^{-(\theta_1+\rho_1) y_1^{\sigma}} dy_1\right]$$

Let : $u_1 = (\theta_1 + \rho_1) y_1^{\sigma}$, $du_1 = \sigma(\theta_1 + \rho_1) y_1^{\sigma-1} dy_1$

$$R_1 = \left[\frac{\rho_1}{(\theta_1 + \rho_1)} \int_0^{\infty} e^{-u_1} du_1\right]$$

$R_1 = \left[\frac{\rho_1}{\theta_1+\rho_1}\right]$ **...(7)**

Case two : When unit A fails and the standby unit B is becomes an active unit, so :

$R_2 = p[X_1 < Y_1, X_2 \geq Y_2]$

$R_2 = p[X_1 < Y_1, mX_1 \geq kY_1]$

Where $X_2 = mX_1$ و $Y_2 = kY_1$

$R_2 = \int_0^{\infty} p(X_1 < Y_1) p(mX_1 \geq kY_1) g(y_1) dy_1$

$= \int_0^{\infty} \left(\int_0^{y_1} f(x_1)\, dx_1\right) \left(\int_{\frac{k}{m}y_1}^{\infty} f(x_1)\, dx_1\right) g(y_1) dy_1$

$R_2 = \int_0^{\infty} (F_{x_1}(y_1)) \left(\bar{F}_{x_1}\left(\frac{k}{m} y_1\right)\right) g(y_1) dy_1$ **..(8)**

substituting equations (1),(2),(3) and (4) in (8), will get as

$$R_2 = \left[\int_0^{\infty} \left(1 - e^{-\theta_1 y_1{}^{\sigma}}\right) \left(e^{-\theta_1 (\frac{k}{m} y_1)^{\sigma}}\right) \sigma\rho_1 y_1{}^{\sigma-1} e^{-\mu_1 y_1{}^{\sigma}} dy_1\right]$$

$$R_2 = \left[\left(\int_0^{\infty} \sigma\rho_1 y_1{}^{\sigma-1} e^{-\left(\theta_1 \left(\frac{k}{m}\right)^{\sigma} + \rho_1\right) y_1{}^{\sigma}} dy_1\right) - \left(\int_0^{\infty} \sigma\rho_1 y_1{}^{\sigma-1} e^{-\left(\theta_1 \left(1 + \left(\frac{k}{m}\right)^{\sigma}\right) + \rho_1\right) y_1{}^{\sigma}} dy_1\right)\right]$$

Suppose that:

$u_2 = \left(\theta_1 \left(\frac{k}{m}\right)^{\sigma} + \rho_1\right) y_1{}^{\sigma}$, $du_2 = \sigma\left(\theta_1 \left(\frac{k}{m}\right)^{\sigma} + \rho_1\right) y_1{}^{\sigma-1} dy_1$

$u_3 = \left(\theta_1 \left(1 + \left(\frac{k}{m}\right)^{\sigma}\right) + \rho_1\right) y_1{}^{\sigma}$, $du_3 = \sigma\left(\theta_1 \left(1 + \left(\frac{k}{m}\right)^{\sigma}\right) + \rho_1\right) y_1{}^{\sigma-1} dy_1$

$$R_2 = \left[\left(\frac{\rho_1}{\left(\theta_1\left(\frac{k}{m}\right)^{\sigma}+\rho_1\right)}\int_0^{\infty} e^{-u_2}\,du_2\right)-\left(\frac{\rho_1}{\left(\theta_1\left(1+\left(\frac{k}{m}\right)^{\sigma}\right)+\rho_1\right)}\int_0^{\infty} e^{-u_3}\,du_3\right)\right]$$

$$= \left[\frac{\rho_1}{\left(\theta_1\left(\frac{k}{m}\right)^{\sigma}+\rho_1\right)} - \frac{\rho_1}{\left(\theta_1\left(1+\left(\frac{k}{m}\right)^{\sigma}\right)+\rho_1\right)}\right]$$

$$R_2 = \left[\frac{\theta_1\rho_1}{\left(\theta_1\left(\frac{k}{m}\right)^{\sigma}+\rho_1\right)\left(\theta_1+\theta_1\left(\frac{k}{m}\right)^{\sigma}+\rho_1\right)}\right] \quad \textbf{...(9)}$$

Now, substituting (7) and (9) in (5) ;we get the reliability function for (1+1) cascade model of Weibull distribution ; R can be written as :

$$R = \left[\frac{\rho_1}{\theta_1+\rho_1}\right] + \left[\frac{\theta_1\rho_1}{\left(\theta_1\left(\frac{k}{m}\right)^{\sigma}+\rho_1\right)\left(\theta_1+\theta_1\left(\frac{k}{m}\right)^{\sigma}+\rho_1\right)}\right] \quad \textbf{...(10)}$$

Numerical Study of Reliability Model

We perform numerical study by calculating reliabilities in the mathematical form R for Weibull Distribution for different values of strength and stress parameters $(\sigma, \theta_1, \theta_2, \rho_1, \rho_2)$.
The results as in the table:

Experiment	*k*	*m*	σ	θ_1	ρ_1	R_1	R_2	R
1	1.1	0.6	1	1	1	0.5000	0.0921	0.5921
2	1.1	0.6	0.25	1	1	0.5000	0.1461	0.6461
3	1.1	0.6	2	1	1	0.5000	0.0428	0.5428
4	1.1	0.6	1	0.25	1	0.8000	0.1003	0.9003
5	1.1	0.6	1	2	1	0.3333	0.0643	0.3976
6	1.1	0.6	1	1	0.25	0.2000	0.0389	0.2389
7	1.1	0.6	1	1	3	0.7500	0.1064	0.8564
8	1.5	0.25	1	1	1	0.5000	0.0179	0.5179
9	1.1	0.75	1	0.25	1	0.8000	0.1132	0.9132
10	1.1	0.99	1	1	3	0.7500	0.1428	0.8928

Discussions

1. When comparing experiment (1) with experiment (2), we notice that the reliability value increases with the decrease in the value of the parameter σ .
2. When comparing experiment (1) with experiment (3), we notice that the reliability value decrease with the increases in the value of the parameter σ .
3. When comparing experiment (1) with experiment (4), we notice that the reliability value increases with the decrease in the value of the parameter θ .
4. When comparing experiment (1) with experiment (5), we notice that the reliability value decrease with the increases in the value of the parameter θ .
5. When comparing experiment (1) with experiment (6), we notice that the reliability value decrease with the decrease in the value of the parameter ρ.
6. When comparing experiment (1) with experiment (7), we notice that the reliability value increases with the increases in the value of the parameter ρ .
7. When comparing experiment (1) with experiment (8), we notice that the reliability value decrease with the increases in the value of $\left(\frac{k}{m}\right)$.

8. When comparing experiments (4) and (7) with experiments (9) and (10), we notice that the reliability value increases with the decrease in the value of $\left(\frac{k}{m}\right)$.

5. Conclusions

1. Reliability is decreasing with increases value of parameter σ.
2. Reliability is decreasing with increases value of parameters θ_1 and θ_2 .
3. Reliability is increasing with increases value of parameters ρ_1 and ρ .
4. Reliability is decreasing with increases value of $\frac{k}{m}$.

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